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EQUIVALENT GRANULAR VOID RATIO AND BEHAVIOUR OF LOOSE SAND WITH FINES

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ABSTRACT

Traditionally, void ratio is being used as a state variable for liquefaction analysis of sand with fines. However, recent publications reported that void ratio is not a good state variable for sand with fines. Thus, equivalent granular void ratio, e^* instead of void ratio, e is proposed to resolve this problem. A single trend for Steady State (SS) data points for sand with fines can then be achieved in e^*-log(p') space irrespective of fines contents, f_c. The single trend is referred to as equivalent granular steady state line, EG-SSL. However, a parameter b, the active fraction of fines in sands force structure, is needed in the conversion to e^* from e. The biggest challenge in using e^* is to get physically reasonable b. There are many approaches namely assumed/back analysis, prediction etc have been seen in literature to obtain b. This paper critically examined these approaches in the light of critical state soil mechanics, CSSM framework. Then the prediction approach is used to examine the applicability of the EG-SSL as an alternative SSL irrespective of fines contents under CSSM framework. The outcome was excellent. Then the prediction approach is used to correlate other void ratio dependent soil behaviours such as cyclic resistance to liquefaction (CR), instability stress ratio (η_{IS}), small strain shear modulus (G) etc. Good correlations were also observed for these behaviours.

INTRODUCTION

Recent publications show that void ratio, e may not be a good parameter for characterizing the behaviour of sand with fines under critical state soil mechanics, CSSM framework. The steady state line, SSL is dependent on fines content. The SSL moved downward in the e-log(p') space with increase in f_c, but the direction of movement eventually reversed at higher f_c. The fines content, f_c at this reversal point is termed as threshold fines content, f_{thre}. Thus, there is a family of SSLs for $f_c \leq f_{thre}$ in e-log(p') space and they must be known to analyze the soil behaviour under CSSM framework. Thus, a large number of tests and time required to characterize the behaviour of sand with fines.

Alternatively, many researcher proposed to used equivalent granular void ratio, e^* as an alternative state to void ratio, e, in attempt to obtain a single trend line for SS data points in the e^*-log(p') space irrespective of f_c provided $f_c \leq f_{thre}$ (Rahman and Lo 2007a; Rahman and Lo 2007b; Rahman and Lo 2008a; Thevanayagam et al. 2002). The single trend line for SS data points for sand with fines was referred to as equivalent granular steady state line, EG-SSL. In the conversion from e to e^*, a parameter b, the fraction of fines that are active in sand force structure, is needed. The biggest challenge in using e^* is to get a physically and theoretically reasonable b value. The most of the b values reported in the literature are either assumed or back analyzed values and it was also assumed that b is constant irrespective of fines contents for a sand with fines. However, some approaches have been seen to correlate these back analyzed b with soil properties (Ni et al. 2004; Thevanayagam 2001). Recently, a prediction formula for b is proposed based on simple input parameters to obtain a single trend line for SS data points in the e^*-log(p') space irrespective of f_c i.e. the EG-SSL (Rahman and Lo 2008b; Rahman et al. 2009).

However, having a single trend line for the SS data points in the e^*-log(p') space does not guarantee that the EG-SSL can be used in the context of the CSSM framework in predicting behaviour of sand with fines. If it does, the question need to answer whether e^* can be used to predict other void ratio dependent soil behaviours such as cyclic resistance to liquefaction (CR), instability behaviour, small strain shear modulus etc irrespective of fines contents.

The objective of this paper is to examine the following issues. First, the effect of b value, obtained from different approaches, on EG-SSL and the theoretical condition to be fulfilled to use the EG-SSL as an alternative SSL irrespective of fines contents. Second, whether the EG-SSL can be used as an alternative SSL to predict undrained behaviour (flow, limited flow and non-flow) of sand with fines or alternatively whether the EG-SSL can be used to predict the SSL for sand with a particular fines content. Third, whether the EG-SSL can be used to define the equivalent granular state parameter, ψ^* in the line with Been and Jefferies (1985) and whether a correlation can be obtained between ψ^* and instability stress ratio, η_{IS} as discussed in Yang (2002) which has many potential application in soil mechanics. Forth, whether the e^* can also be used to predict other void ratio dependent soil behaviours such as cyclic resistance to liquefaction (CR), instability behaviour, small strain shear modulus etc irrespective of fines contents.

LITERATURE

Equivalent Granular Void Ratio

The predecessor of e^* is intergranular void ratio, e_g. The concept e_g was first used by Mitchell (1976) to determine the inactive clay content on soil structure. The core idea in the formulation was to assume inactive clay content as void. One year, later Kenney (1977) found, soil containing clay minerals and water less than 40% to 50% of total volume showed a residual strength is equal to that of the granular mineral (quartz) only i.e. clay and water is inactive in granular force structure. Troncoso and Verdugo (1985) also found similar out come from cyclic triaxial experiment on the tailing sand with 30% fines. Although these results support the concept of intergranular void ratio, Kuerbis et al. (1988) may be the first researcher that used intergranular void ratio as the basis for comparing undrained shear strength behaviour. He suggested that fines particles simply be occupying gaps in the sand skeleton and therefore, the measured behaviour is controlled by sand skeleton only. Thus, neglecting the volumes of fines he calculated intergranular void ratio as

$$e_g = \frac{V_T G_S \rho_W - (M - M_{silt})}{(M - M_{silt})} \tag{1}$$

Where, e_g is intergranular void ratio. He observed almost similar soil strength behaviour at the same intergranular void ratio. Georgiannou et al. (1990) also found similar out come for clayey sand (Ham river sand) and proposed intergranular void ratio as

$$e_g = \frac{\text{Volume of voids} + \text{volume of clay}}{\text{volume of granular phase}} \tag{2}$$

He observed that the effective stress paths in UC tests were almost identical for similar intergranular void ratio. A simplified formulation of intergranular void ratio was given by Thevanayagam (1998) as:

$$e_g = \frac{e + f_c}{1 - f_c} \tag{3}$$

where, e is void ratio, f_c fines content. Equation (3) is based on assumption that the all of the fines in sand-fines mixture can be approximated as void space, i.e. their contribution to the force structure can be neglected. It should be noted that the intergranular void ratio, e_g was also being referred to as void ratio of granular phase (Mitchell 1976), skeleton void ratio (Kuerbis et al. 1988), granular void ratio (Georgiannou et al. 1990). However, the work of Pitman (1994), among other, unambiguously demonstrated that e_g can function as an alternative to e provided that f_c content is low relative to the void space formed by the hoist sand.

At higher fines content (relative to the void space), the fines begin to participate in the force structure. Therefore, Thevanayagam et al. (2002) proposed the use of equivalent granular state parameter, e^* defined by Equation (4) below as a better alternative state to e:

$$e^* = \frac{e + (1-b) f_c}{1 - (1-b) f_c} \tag{4}$$

where, b represents the fraction of fines that are active in force structure. The basic assumption of e^* behind Equation (4) requires $1 \geq b \geq 0$. The e^* was also being referred to as, equivalent inter-granular contact void ratio (Thevanayagam et al. 2002), corrected intergranular void ratio (Yang et al. 2006b), equivalent inter-granular contact index void ratio (Thevanayagam 2007), equivalent granular void ratio (Rahman and Lo 2008b; Rahman et al. 2008).

Challenges with the Parameter b

By setting $b = 0$, the definition of e^* reduced to that of e_g. This implies, $b = 0$ (which yield $e^* = e_g$) is an adequate approximation at low fines content, but the non-zero value of b should be considered at higher fines content. A corollary is that b is a function of f_c, and this is also consistent with the 'fines-in-sand' soil fabric. However, most of the b-values reported in various publications were independent of f_c i.e. constant. Furthermore, they were deduced by back-analysis of extensive triaxial tests covering a range of fines contents. Therefore, such a b value is in fact an average value for the range of fines contents covered in a particular publication and was selected to achieve a desired relationship for a range of f_c. This sometimes led to unreasonable b values such as Ni et al. (2004) reported a very high b value of 0.7 being achieved for a low f_c of 9% and a negative b value of -0.8 for 9% fines content. Sometime, it can lead to narrow trend but also unreasonable correlation. For example, the data from Zlatovic and Ishihara (1995) was used in many literature to achieve a single trend line in the e^*-log (p') space with a $b = 0.25$ as shown in Fig. 1 (Ni et al. 2004). The trend line for Toyoura sand with 5% to 30% (shown in blue solid line) is different than for Toyoura sand with 0% fines i.e. clean Toyoura sand (shown in red dotted line) i.e. the EG-SSL for sand with 0% fines content is not the same as sand with other fines contents. Therefore, there are some initial states in the e^*-log (p') space (as shown in Fig. 1) that might indicate contractive behaviour for clean sand and dilative behaviour for sand with fines. This is not consistent with CSSM framework and does not testify the EG-SSL as an alternative SSL irrespective of fines contents. Again, it is not possible to achieve a single trend line (with small scatter) around sand with 0% fines content with a constant b irrespective of fines contents. However, a single trend around sand with 0% fines content can be achieved with a variable b with fines contents. This is also in support that b is a function of f_c.

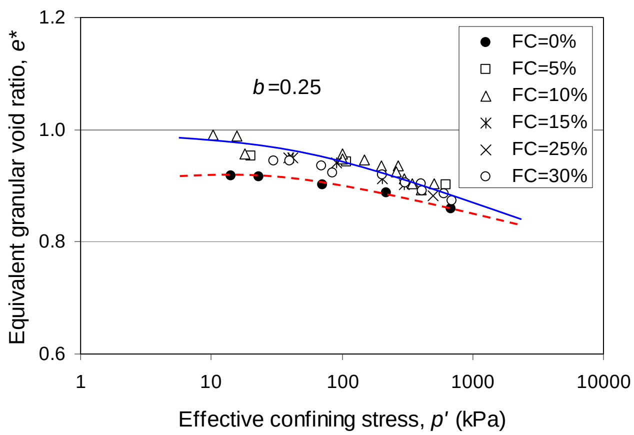

Fig. 1: The EG-SSL for Toyoura sand with fines with a constant b=0.25 after Zlatovic and Ishihara (1995).

<u>Prediction of the Parameter *b*</u>

Rahman and Lo (2008b; 2008) , based on a re-analysis of the experimental data of McGeary (1961) on binary packing studies, concluded that b is a function of f_c and particle diameter ratio, χ, i.e. $b = F(\chi, f_c)$. Furthermore, the functional relationship has to possess certain mathematical attributes. A semi-empirical equation expressed as Equation (5) was proposed.

$$b = \left[1 - \exp\left(-\mu \frac{(f_c / f_{thre})^{n_b}}{k}\right)\right] \times \left(r \frac{f_c}{f_{thre}}\right)^r \quad (5)$$

where, $r = (D_{10}/d_{50})^{-1} = d_{50}/D_{10}$, $k = (1 - r^{0.25})$. D_{10} = size of sand at 10% finer and d_{50} = size of fines at 50% finer.

To further simplifies the equation, one can assign $n_b = 1$ and have μ as the calibration constant. Rahman and co-workers (Rahman and Lo 2008b; Rahman et al. 2009) showed that a single value of $\mu = 0.30$ can be approximated for a large number of data sets. Thus, the Equation (5) can be simplified to:

$$b = \left[1 - \exp\left(-0.3 \frac{(f_c / f_{thre})}{k}\right)\right] \times \left(r \frac{f_c}{f_{thre}}\right)^r \quad (5a)$$

The prediction of f_{thre} is another issue to be addressed before Equation (5a) can be used. Rahman and Lo (2008b) suggested the use of the following Equation based on calibration with ten datasets.

$$f_{thre} = A_{TFC}\left(\frac{1}{1 + e^{\alpha - \beta\chi}} + \frac{1}{\chi}\right) \quad (6)$$

The coefficient of A is the asymptotic value of 0.40. The other two parameters α and β are determined by curve fitting and this gave $\alpha = 0.50$ and $\beta = 0.13$. Rahman and Lo (2008b) verified the concept of a single EG-SSL by examining ten data sets from seven different countries around the world. Equation (6) and (5a) was used to calculate the b value which in turn enables the conversion of e to e^* by Equation (4).

COMPARISION OF DIFFERENT APPROACHES TO OBTAIN *b*

This section discussed about the advantages and disadvantages of back analyzed and predicted b values. The back analyzed b value reported in literature is usually a constant value irrespective of fines contents. The main advantage of the back analyzed b is that a good correlation can be achieved. However, some 'out of trend' is also expected in this case as shown in Fig. 1. To avoid this problem, the data point can also be back analyzed for different b values for different fines contents. However, to apply this approach in practical purpose one should have a large number of data for different fines contents. This is the main disadvantage of back analysis process.

On the other hand the prediction Equation (5) is a generalized equation and it is expected to give good correlation for different sand with fines. However, the parameter μ and n_b need to calibrate with the pre-existing data sets. It is also a back analysis process under some physical constrains and expected to give better correlation than a constant b. The Equation (5a) is a true prediction formula with further assumption and can be used with simple input parameters such as particle size ratio and fines content.

Thus, the b parameter can be obtained by different approaches to achieve a single correlation for SS data points. To compare the performance of these different approaches, the root-mean square-deviation, RMSD around the trend line is taken as benchmark. The trend line with smaller RMSD means better correlation and vice versa. The Table 1 shows the performance of different approaches in terms of RMSD. The different approaches in Table 1 represented by the following numbers:

1. A constant b irrespective of fines content
2. b increasing with fines content
3. μ and n_b is variable in Equation (5)

4 μ is variable and n_b =1 in Equation (5)
5 Equation (5a)

Six data sets have been used in this comparative study. The detail of sand and fines used on those studies are given in the Table 1. The *b* values were obtained by the different approaches as outlined above and the correlation of individual data sets around the trend line is presented in terms of RMSD.

Table 1. RMSD around trend line for different data sets with different *b* by different approaches

Sources	Sand				Fines			r=1/χ	f_{thre}	RMSD for different approaches				
	Name	D_{50}	D_{10}	Cu	Name	D_{50}	Cu			1	2	3	4	5
Rahman (2009)	Sydney	0.270	0.220	1.26	Majura	0.005	12.5	0.025	0.40	0.030	0.016	0.016	0.016	0.016
Thevanayagam et al. (2002)	OS00	0.250	0.160	1.69	Silica	0.010	7.50	0.063	0.36	0.019	0.019	0.022	0.027	0.027
Haung et al. (2004)	Mai Liao	0.123	0.080	1.75	Mai Liao	0.044	2.79	0.550	0.41	0.027	0.020	0.031	0.032	0.028
Ni et al. (2004)	Allu-vium	0.778	0.209	5.63	Allu-vium	0.038	5.43	0.182	0.30	0.014	0.011	0.011	0.011	0.015
Yang et al. (2006a)	Hokk-sund	0.440	0.225	2.25	Chen-gbei	0.032	2.32	0.142	0.30	0.033	0.031	0.031	0.032	0.032
Zlatovic and Ishihara(1995)	Toyo-ura	0.170	0.116	1.61	Toyo-ura	0.010	6.08	0.086	0.33	0.022	0.015	0.015	0.032	0.051

The RMSD in column 1 was achieved by a constant *b* irrespective of fines contents. However, the RMSD in column 2 was achieved by a variable *b* (increasing) with fines contents. In most cases, the RMSD in column 2 is less than the column 1. This indicates a variable *b*, consistent with binary packing theory, is a better approximation than a constant *b*. These two approaches were purely based on back analysis.

The RMSD in column 3 and 4 were also obtained by back analysis but under some physical constrained imposed by equation 5. The RMSD in column 3 was obtained by considering μ and n_b are variable in Equation (5). On the other hand, the RMSD in column 4 was obtained by considering μ is a variable and n_b = 1 in Equation (5). The RMSD in column 3 is smaller that column 4.

The RMSD in column 5 was achieved by the predicted *b* using Equation (5a). This is the only true prediction approach with simple input parameters such as particle diameter ratio and fines content. The RMSD values were higher than column 2, 3, 4 but comparable to column 1 (constant *b*). The column 5 showed higher RMSD for two data sets (Thevanayagam et al. 2002; Zlatovic and Ishihara 1995) than column 1. It is some what expected that prediction approach might show higher scatter than back analysis approach. However, the scatter for the data set Thevanayagam et al. (2002) was small and it should be consider as a correlation. But the scatter for the data sets of Zlatovic and Ishihra (1995) was high and may not be considered as correlation. However, the correlation with constant *b* is still affected by the problem as explained in Fig. 1. It should be noted that in this paper hereafter the *b* are always obtained by prediction using Equation (5a) & (6) and refer to as the 'prediction approach'.

EXPERIMENTATION

A series of monotonic triaxial tests have been done on sand with a range of fines content: 0%, 15%, 20%, and 30%. The details of testing material and testing procedure are given below.

<u>Tested Material</u>

The host sand was Sydney sand, a medium quartz clean sand (SP) and it's index properties can be found in Lo et al. (1989). The fines is a specially designed low plasticity fines (PI=27, LL=54) with a uniformity coefficient 12.56. It is composed of 2/3 of well-graded silt from the Majura River and 1/3 commercial kaolin.

<u>Experimental Procedure</u>

A strain controlled triaxial loading system with fully automated data logging facilities was used for this study. Axial load was measured with an internal load cell. The axial deformation was measured by two independent means: a pair of internal LVDTs mounted directly across the platens and an external LVDT. The former was used in the early stage of shearing whereas the latter was used at large deformation. Cell pressure was controlled by a large capacity Digital Pressure Volume Controllers (DPVC). The pore pressure line was connected to a small capacity DPVC for controlling back pressure (and measuring the volume change) at the consolidation stage and for imposing an undrained condition and measuring the resultant pore pressure response during shearing. Two pressure transducers were also used to verify pore pressure equilibrium.
A modified moist tamping method was used for specimen preparation to ensure uniformity of the specimen. To

accurately control the void ratio, a total of 10 layers of predetermined quantities of moist soil were worked into a prescribed thickness. Details of specimen preparation method can be found at Rahman et al. (2008). Enlarged end platens with free ends, as describe by Lo et al. (1989), was used to minimize end restraint. Liquid rubber technique was used to minimize bedding and membrane penetration error, and also to ensure even seating at the top platen.

Saturation of the specimen was accomplished in two stages. Stage-I consisted of carbon dioxide, CO_2 percolation for at least 20 minutes followed by vacuum flushing with a low head. Stage-2 was back pressure application to achieve a Skempton B-value of at least 0.98. Special efforts were made to measure the void ratio accurately, which is an essential requirement for this study.

APPLICATION OF EQUIVALENT GRANULAR VOID RATIO

Equivalent Granular Steady State Line, EG-SSL

The SS data points in the e^*-log(p') space are shown in Fig. Fig. 2. The Equations (6), (5a) and (4) were used to calculate e^*. Evidently, all SS data points in e^*-log(p') space followed a single trend i.e. a single EG-SSL was obtained. However, this EG-SSL is not a straight line, but rather a curve which is consistent with many publications (Bobei and Lo 2005; Wang et al. 2002). Thus, the curvature should be born in mind though it is usually referred to as line. The EG-SSL can be presented by the following Equation

$$e^* = 0.908 - 0.0266\left(\frac{p'}{p_a}\right)^{0.7} \tag{7}$$

where, p' = mean effective stress, p_a = atmospheric pressure. The RMSD around this trend line is 0.016 as presented in Table 1.

Prediction of SSL for sand with 10% fines

The equivalent granular steady state line, EG-SSL can be used to predict the SSL of sand with particular fines content without performing extensive number of triaxial tests on sand with that particular fines content. In earlier section, it is mentioned that the SSL of clean sand and the EG-SSL of sand with fines essentially merge in a single trend. Hence, if the SSL of clean sand or the EG-SSL of sand with fines is known, then the SSL i.e. the void ratio, e on the SSL for sand with particular fines content can be predicted from the equivalent granular void ratio, e^* on the EG-SSL. The calculation of void ratio, e from the equivalent granular void ratio, e^* can be done by rearranging the Equation (4):

$$e = e^* - (1 + e^*) \times (1 - b) \times f_c \tag{8}$$

where, e^* = equivalent granular void ratio = void ratio on EG-SSL. The b for a particular fines content, f_c can be found without triaxial tests using parameters such as r, f_c. For example, the EG-SSL for Sydney sand with fines i.e. the Equation (7) is used to predict the SSL for Sydney sand with 10% fines. The predicted SSL of sand with 10% fines is then compared with the data extracted from Bobei and Lo (2005) and Bobei et al. (2009). The Fig. 3 shows that the triaxial test data fall over the predicted SSL.

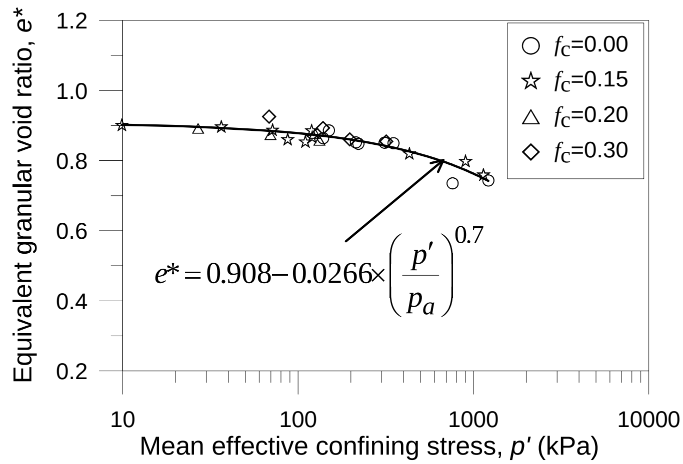

Fig. 2. The EG-SSL for Sydney sand with 30% fines.

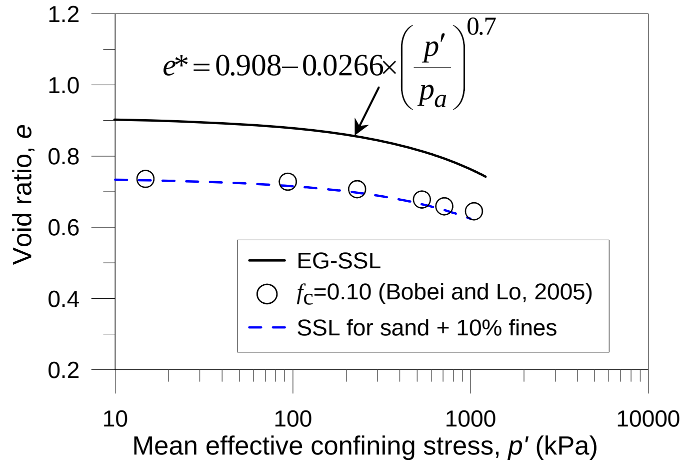

Fig. 3. The prediction of SSL for Sydney sand with 10% fines

The above example highlights an excellent opportunity to estimate the SSL for sand with a particular fines content from a know SSL either for clean sand or for sand with another fines content. For example if the SSL for sand with 15% fines content is known and one may like to know the position of the SSL for sand with 25% fines content in the e-log(p') space. In this particular case, the back analysis processes do not work due to lack SS data for sand with 0% fines content i.e. clean sand. But, the prediction process can be used to estimate the SSL as following-

- the b for sand with 15% fines content can be calculated using the Equations (6) & (5a) to achieve the EG-SSL
- then, the b for 25% fines content can be calculated from the Equations (6) & (5a) to obtain the SSL for sand with 25% fines content in the e-log(p') space from the EG-SSL.

This advantage only can be taken from the 'prediction approach'.

Prediction of undrained behaviour

In order to evaluate the EG-SSL for sand with a wide range of $f_c \le f_{thre}$, the initial states (i.e. just prior to shearing) of all tests were plotted as data point in Fig. 4. The EG-SSL in e^*-log(p') space were also plotted in this figure. The data points for tests with flow behaviour were all located well above the EG-SSL. Tests with non-flow were represented by data points located well below the EG-SSL. The data points for tests with limited-flow behaviour were plotted around the EG-SSL. This demonstrated that the EG-SSL, in conjunction with the initial states, can be used within the CSSM framework in predicting the overall undrained behaviour patterns.

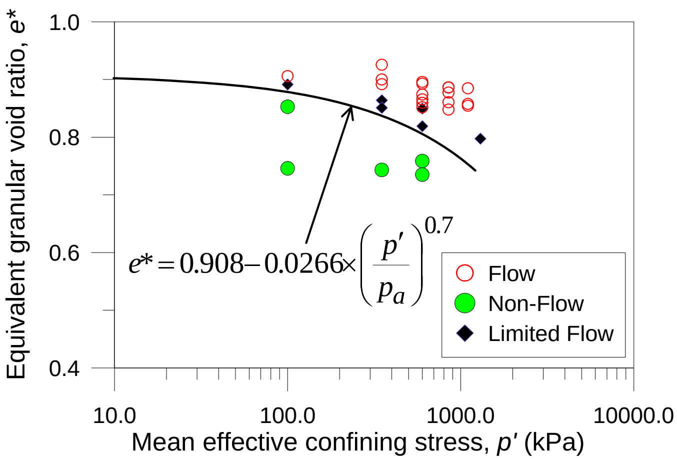

Fig. 4. Initial condition for undrained tests with different behaviour (flow, non-flow, limited flow)

CORELATION FOR CYCLIC RESISTANCE, INSATABILITY, SHEAR MODULUS

It may be possible to achieve a single trend line for SS data points i.e. the EG-SSL in e^*-log(p') space for a verity of sand with fines using the prediction formula for b. The EG-SSL can then be used to predict the SSL for any particular fines content or may be used to predict undrained behaviour of sand with fines. Now, one may ask whether this prediction approach can be used to achieve single trend for other void ratio dependent soil behaviours such as cyclic resistance, Instability or small strain shear modulus etc. If so, the correlation can be used to predict these behaviours for any particular fines content. The details of these evaluations are discussed below.

Correlation of Cyclic Resistance to Liquefaction, CR

To evaluate whether a single correlation between cyclic resistance to liquefaction, CR and e^* can be achieved, a data set was extracted from Vaid (1994). Vaid (1994) performed cyclic triaxial tests on Brenda 20/200 sand with non-plastic fines, and the fines contents ranges from 0 to 21%. Brenda sand is an angular tailing sand. The spread of data points in e - CR space was about 0.34 in terms of e. The e was then converted to e^* using prediction Equations (6), (5a) and (4). The input for the prediction approach was r = 0.10 and f_c. The data point was then plotted in e^* - CR space as shown in Fig. 5. An essentially single correlation between e^* and CR, around the data points for sand with 0% fines content was obtained. The RMSD of the correlation was 0.0255.

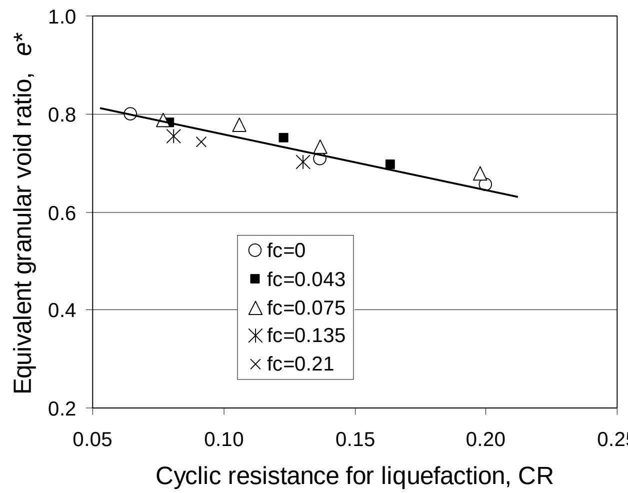

Fig. 5. Correlation between e and CR with the b predicted by Eqns (6), (5a) & (4), after Vaid (1994)*

Correlation to Instability Stress Ratio, η_{IS}

Very loose sand exhibit a peak strength at small strain before collapse to a lower mean effective stress and lower strength at large strain. The stress ratio, η at the peak strength is the onset of instability and termed as instability stress ratio, η_{IS} hereafter. Yang (2002) demonstrated that there is a correlation between η_{IS} and ψ in the η_{IS} - ψ space that has potentially many applications in soil mechanics (Imam et al. 2002). Considering the complexity with the presence of fines, one has to define equivalent granular state parameter, ψ^* considering the EG-SSL as alternative SSL to achieve such a relation for sand with fines. The details can be found in another paper presented in this conference (Baki et al. 2010).

Equivalent Granular State Parameter. Along the line of Been and Jefferies (1985), Rahman and Lo (2007a) introduced the concept of equivalent granular state parameter, ψ^* defined by:

$$\psi^* = e^* - e^*_{SS} \tag{9}$$

where, e^*_{SS} = equivalent granular void ratio at the EG-SSL. The definition of ψ^* is schematically illustrated in Fig. 6. The initial value of ψ^* (before undrained shearing) is denoted by $\psi^*(0)$ here after. It is pertinent to note that having the initial state plotted above the EG-SSL is equivalent to having $\psi^*(0) > 0$, and that $\psi^*(0) < 0$ means having the initial state located below the EG-SSL. Thus the definition of Equation (9) is consistent with the findings of the previous verification.

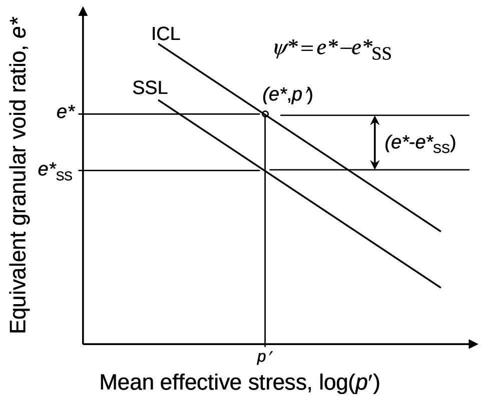

Fig. 6. Definition of equivalent granular state parameter, ψ^*

Correlation Between η_{IS} and $\psi^*(0)$. The Fig. 7 shows that there is a correlation exists between η_{IS} and ψ^* in the η_{IS} - ψ^* space for Sydney sand with fines. Thus, the relation can be used to estimate onset instability under monotonic loading at any initial condition provided that $\psi^* > 0$. Many literatures reported that there is a link between monotonic and cyclic instability (Gennaro et al. 2004; Hyodo et al. 1994; Lo et al. 2008; Mohamad and Dobry 1986; Vaid and Sivathayalan 2000; Yamamuro and Covert 2001). Thus, this relationship might be used to predict cyclic instability provided that cyclic instability triggered in compression side. The detail of this is explained in another paper submitted to this conference (Baki et al. 2010).

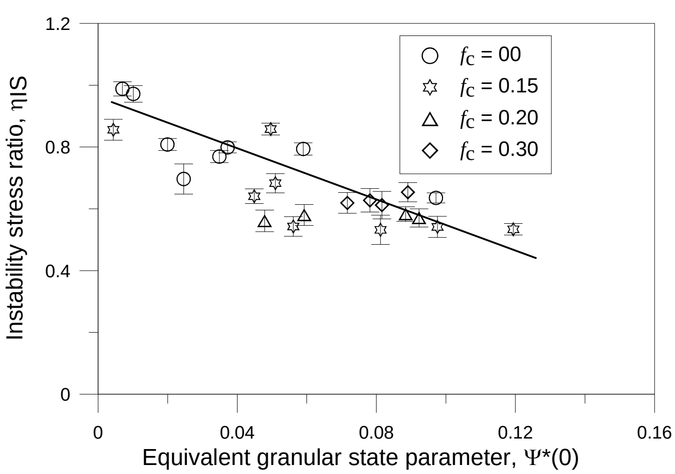

Fig. 7. The relation between instability stress ratio, η_{IS} and equivalent granular state parameter, ψ^*

Correlation to small strain shear modulus

It is well established that small strain shear modulus of soil, an important parameter for dynamic response analysis, depends on mean effective confining stress and void ratio i.e. $G = Af(p', e)$. As void ratio, e is not a good state variable for sand with fines, G for sand with fines is expected to be affected by the presence fines. It is found in literature that G value move downward with increase in fines content in e-G space (Iwasaki and Tatsuoka 1977; Thevanayagam and Liang 2001). To evaluate whether the prediction formula for e^* using Equation (6), (5a) and (4) can develop a single trend in e^*-G space, a set of data was extracted from Thevanayagam and Liang (2001). The secant shear modulus $G_{0.05}$ (measured at 0.05% axial strain) for OS00 sand with non-plastic crushes silica fines move downward with increase in fines content. For the prediction approach, the only input parameter was diameter ratio, r = 0.063 and the f_c. The all data points for OS00 sand with different fines contents come to a single trend around the data points for sand with 0% fines content in e^*-G space. Thus, this relation can be used to predict/estimate G for sand with any particular fines content.

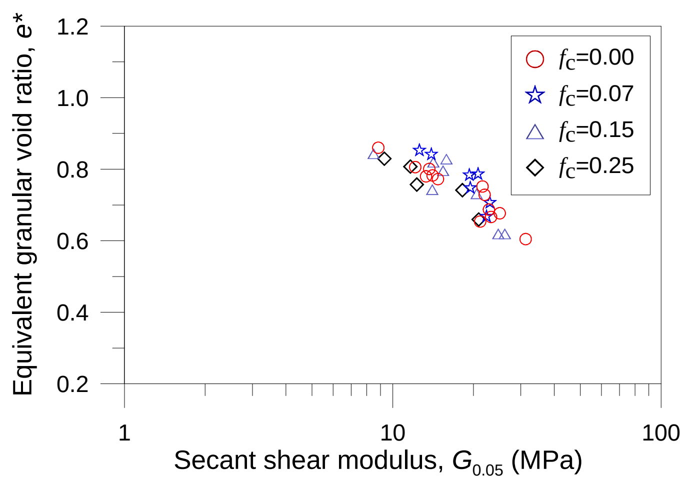

Fig. 8. The relation between secant shear modulus, G and equivalent granular void ratio, e^*.

CONCLUSIONS

A series of undrained triaxial tests on Sydney sand with up to 30% fines contents have been done and five other data sets on steady state behaviour have been analysed to compare two different approaches, 'back analysis' and 'prediction approach', obtain b i.e. the equivalent granular void ratio, e^* for sand with fines. The study reveal that the back analysis approach may sometime give better overall correlation than prediction approach; however, it is found that the overall correlation is not always a desirable criteria under critical state soil mechanics, CSSM framework. On the other hand, an acceptable correlation can be obtain by the prediction approach; however the main advantage over the back analysis is that it can be used as a prediction tool based on very simple input parameter such particle size ratio, r and fines content, f_c.

Then, the prediction method was used to obtain the equivalent granular steady state line, EG-SSL to investigate whether the EG-SSL can be used as an alternative SSL for sand with fines. The main findings of this study are summarized below:

- The prediction approach gives an opportunity to obtain the EG-SSL for sand with fines from simple input parameters such as particle diameter ratio and fines contents.
- The EG-SSL obtained from the prediction approach can be used to predict undrained behaviour such as flow, non-flow or limited flow behaviour under CSSM framework.
- The prediction approach can used to obtain single correlation for cyclic resistance to liquefaction, instability stress ratio, small strain shear modulus and these relation can be used to estimate unknown behaviour for sand with a particular fines content.

It is noted that the data base used in this study are essentially for non-plastic to low-plastic fines and one should not extrapolate the findings to plastic fines. The concept of equivalent granular void ratio is valid for gap graded sands and particle shape should be round.

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